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Capacity Analysis of Traffic Flow Over a Single-Lane Automated Highway System☆

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We calculate bounds on per-lane Automated Highway System (AHS) capacity as a function of vehicle capabilities and control system information structure. We assume that the AHS lane is dedicated for use by fully automated vehicles. Capacity is constrained by the minimum inter-vehicle separation necessary for safe operation. A methodology for deriving the safe minimum inter-vehicle separation for a particular safety criterion is presented. The inter-vehicle separation, which depends on the vehicle braking capability, control loop delays and operating speed, is then used to compute site-independent upper bounds on AHS capacity for a given mix of vehicle classes. The sensitivity of the capacity with respect to the degree of inter-vehicle cooperation, check-in policies (governing minimum acceptable vehicle-braking capability), highway speed limits, and lane-use policies (governing the sharing of a lane by multiple vehicle classes) is also investigated.

Keywords: Automated highway system; Highway capacity; Traffic safety; Individual vehicle; Platoon; Game theory

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1. INTRODUCTION

The goal of an Automated Highway System (AHS) is to significantly increase safety and highway capacity, without having to build new roads, by introducing automation to both the vehicles and the highway infrastructure. Various alternatives have been proposed for organization of automated traffic on an AHS. In this paper, we analyze the effectiveness of four distinct AHS designs in terms of their corresponding maximum achievable per lane capacity.

Calculating the expected throughput for a proposed AHS design is a challenging task in general [1–3,21]. In this paper we consider the special case of an *AHS pipe*, a single-lane AHS without entrances (except its beginning) and exits (except its end). We estimate the maximum possible steady state flow in vehicles per lane per hour through such an AHS pipe for various AHS operating speeds. The flow through an actual AHS lane will always be less than our steady-state estimate due to transient [3,16] effects such as lane changing, demand, etc. Since the pipe estimate is an upper bound on the actual lane flow, the estimate at every AHS operating speed is called the *pipeline capacity* at the speed.

Under equilibrium conditions, traffic flow (as also pipeline capacity) is related to speed and density by

$$\textit{pipeline capacity} = \textit{density} \times \textit{speed},$$

where density is the reciprocal of the combined inter-vehicle spacing and vehicle length at the specified speed. The relationship between spacing and speed on regular highways is set by human driving habits. For an AHS this relationship is set by design. Furthermore, we believe that the relationship between inter-vehicle spacing and speed has a significant impact on the safe operation of an AHS. Safety is typically encoded by criteria relating to the possible collisions between the vehicles. On an AHS pipe, each vehicle only has to worry about the possibility of colliding with the vehicle ahead of it; collisions with the vehicle behind are taken care of by symmetry. Different *safety criteria* give rise to different *separation policies*. For example, the *brick wall safety criterion* requires that a vehicle does not collide with the vehicle ahead, even if the latter stops instantaneously. This safety criterion will not be considered here, as it is too stringent. In this paper we consider two weaker safety criteria. The first is the *hard braking safety criterion*, which requires that:

If a vehicle applies maximum braking until it comes to a stop, the following vehicle should be able to stop without colliding with it.

Such a hard braking disturbance may arise on an AHS in response to an obstacle or a vehicle malfunction. Theoretical justification for this safety criterion is given in Section 2. Vehicles following the separation policy dictated by the hard braking safety criterion are referred to as *individual vehicles*. It can be shown [4] that this safety criterion results in a significant reduction of the probability of inter-vehicle rear-end collisions compared to manual operation of vehicles on a conventional highway.

The second safety criterion considered here is the *low relative velocity safety criterion*, which requires that:

If a vehicle applies maximum braking and the following vehicle collides with it, the relative velocity at initial impact should be small.

Theoretical justification for this safety criterion is given in [17]. For a discussion on tolerable relative velocities at impact see [18]. Assuming that initially the speed of both vehicles is the same, it can be shown [17] that the low relative velocity safety criterion can be met if the vehicles are either far apart or close enough to one another. In the former case the vehicles have time to stop before they collide while in the latter they collide very quickly and hence the relative velocity at impact is small. This observation gives rise to the separation policy known as *platooning*. On an AHS that supports platooning, vehicles travel in closely spaced groups (*platoons*) of up to twenty vehicles with intra-platoon separations of the order of 1 to 4 m. Platoons are isolated from each other by larger distances in order to avoid inter-platoon collisions. As we shall see platooning can substantially increase the capacity of the AHS, at the cost of small relative velocity intra-platoon collisions. It should be noted that small intra-platoon spacings guarantee low impact velocities only in the first collision. There is little study regarding frequency and severity of the possible subsequent collisions. Refer to [8,20] for some initial results.

The pipeline capacity at each speed is estimated for each separation policy by using the appropriate safety criterion. The pipeline capacity at the specified speed is then computed by using the equilibrium flow equation already described.

The results presented here: (i) determine the spacing values needed to meet the safety criteria, (ii) estimate pipeline capacity, and (iii) determine the relationship among pipeline capacity, highway and vehicle parameters. In particular, we investigate the sensitivity of AHS pipeline capacity to the following factors:

(1) Separation policy: Individual vehicles only or platooned operation.
(2) Inter-vehicle cooperation: We study three levels of cooperation for individual vehicles.
    (a) *Autonomous*: Vehicles do not communicate with each other.
    (b) *Low cooperation*: Vehicles communicate only during maneuvers and emergencies (e.g., hard braking).
    (c) *High cooperation*: Vehicles continuously exchange state information such as speed and acceleration, in addition to maneuver coordination messages and emergency warnings.

    An AHS that supports platooning is assumed to be high cooperative within the same platoon and low cooperative between different platoons.
(3) AHS operating speed.
(4) Vehicle mix: We consider three classes of vehicles (passenger vehicles, buses, and trucks) and investigate how the percentage of the different classes affects capacity.
(5) Platoon size (for an AHS that supports platooning).
(6) Vehicle braking capability.
(7) Dynamic safe spacing adjustment based on real-time estimation of braking capability.

The results of our investigation can be used as guidelines for selecting between different proposed AHS designs (in terms of separation policy and cooperation), and for policy making (e.g., selecting speed limits, metering policies to limit the range of deceleration capabilities, allocation of vehicles to lanes according to their class, etc.).

The paper is organized in four sections. Section 2 describes the methodology for obtaining inter-vehicle spacing and pipeline capacity. The results for different AHS designs are presented in Section 3. Section 4 summarizes the results and discusses future research.

2. METHODOLOGY

2.1. Spacing Requirements for Individual Vehicles

Consider two vehicles A and B (Fig. 1) traveling in the same direction and in the same lane. Assume the vehicles have lengths L_A and L_B and let x_A and x_B denote their positions with respect to a fixed reference on the road. Following [6,7] and motivated by experimental testing of the brake actuation systems [7], developed at the California Partners for Advanced Transit and Highways (PATH) Program, we model the longitudinal dynamics of a vehicle by a second order system with a pure time delay δ and a first order lag τ (representing lumped sensing, actuation, and processing delays and lags). Let $D = x_B - x_A - L_B$ denote the spacing between the vehicles. Then, the evolution of the system A–B can be described by a state vector, $x = [\dot{x}_A, \ddot{x}_A, D, \dot{D}]^T$, with dynamics:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1/\tau & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1/\tau \\ 0 \\ 0 \end{bmatrix} u(t-\delta) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} d(t), \quad (1)$$
$$x(0) = x^0,$$

where u is the desired acceleration input applied by the controller of vehicle A, $\dot{x}_A$ and $\dot{x}_B$ are the longitudinal velocities of the two vehicles and $\ddot{x}_A$, $d \equiv \ddot{x}_B$ are the corresponding accelerations. We assume that the longitudinal controller for vehicle A has access to full state information, $\dot{x}_A, \ddot{x}_A, D, \dot{D}$ through appropriate sensors. The acceleration of vehicle B, (d), cannot be measured directly by the

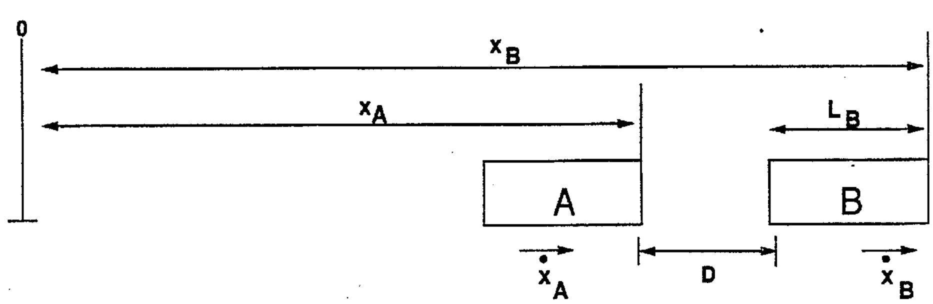

FIGURE 1 Vehicle following scenario.

on-board sensors of vehicle A and is thus treated as an unmeasured disturbance. The vehicle dynamics are constrained by the engine, tire, and road conditions. More specifically, the accelerations and speeds of both vehicles are bounded by

$$x(t)=\{x\in\Re^4 | \dot{x}_A\in[v^A_{\min}, v^A_{\max}],\ \ddot{x}_A\in[a^A_{\min}, a^A_{\max}],\ \dot{x}_B\in[v^B_{\min}, v^B_{\max}]\},$$
$$\dot{u}(t)\in[j^A_{\min}, j^A_{\max}],\qquad d(t)\in[a^B_{\min}, a^B_{\max}].$$

For highway operation, it is assumed that vehicles do not go backwards. Therefore

$$v^A_{\min}=v^B_{\min}=0.$$

Based on this model we derive spacing requirements for individual vehicles needed to guarantee that no collisions are possible on the AHS pipe. The analysis presented here assumes no communication of state information between the vehicles and negligible pure delays ($\delta=0$). The "no collision" requirement for vehicle A can be encoded by a cost function

$$J(x^0,u,d)=-\inf_{t\geq 0} x_3(t).$$

If for a given initial condition x^0 and a given choice of u and d, $J(x^0,u,d)\leq 0$, then vehicle A will not collide with vehicle B. We would like vehicle A to remain safe in this sense whatever B decides to do. We therefore seek the worst possible action of vehicle B and the best possible action of vehicle A, i.e., a pair (u^*,d^*) such that

$$J(x^0,u^*,d)\leq J(x^0,u^*,d^*)\leq J(x^0,u,d^*). \tag{2}$$

In the language of game theory, such a choice of inputs is called a saddle solution to the two-player, zero-sum game between u and d over cost function J. For the above model consider the candidate saddle solution:

$$u^*(t)=\begin{cases} x_2^0-|J^A_{\min}|t & \text{if } t\leq T_1,\\ a^A_{\min} & \text{if } t>T_1,\end{cases}\qquad d^*(t)=\begin{cases} a^B_{\min} & \text{if } t\leq T_2,\\ 0 & \text{if } t>T_2,\end{cases}$$

where T_1 is the time when the acceleration of vehicle A reaches a^{A}_{min} under $u^*(t)$ and T_2 the time when vehicle B stops under a^{B}_{min}. The candidate saddle solution simply requires both vehicles A and B to brake as hard as possible. In can be shown that:

LEMMA: *(u^*, d^*) is globally a saddle solution for cost $J(x^0, u, d)$.*

Refer to [9] for further details regarding the game theoretic formulation and computation of the saddle solution for the vehicle following problem. This lemma can be used to calculate the optimum cost $J^*(x^0)$ for a given initial condition x^0. Moreover, we can distinguish safe states (i.e., $J^*(x^0) < 0$) from unsafe ones (i.e., $J^*(x^0) > 0$) and determine the boundary between them (i.e., $J(x^0) = 0$). Note that for safe states, vehicle A is guaranteed not to collide with vehicle B as long as it starts decelerating (applies $u = u^*$) whenever $J^*(x(t)) = 0$. For unsafe states, on the other hand, there exist actions of vehicle B (in particular $d = d^*$) for which a collision between vehicles A and B is unavoidable, whatever vehicle A chooses to do.

This argument provides theoretical justification for the hard braking safety criterion and allows us to calculate the minimum spacing needed to satisfy it. Using $J^*(x^0)$ this minimum safe spacing can be encoded as a function:

$$S : (\dot{x}_A, \ddot{x}_A, \dot{D}) \mapsto D.$$

The function can be analytically calculated and is parametrized by the vehicle parameters a^{A}_{min}, a^{B}_{min}, j^{A}_{min}, and τ. It will be used in Section 3 to derive safe spacing requirements, and thereby pipeline capacity for individual vehicles and inter-platoon operation.

2.2. Spacing Requirements for Platoons

The state of the art longitudinal control laws proposed for intra-platoon operation [6] are significantly different from those for individual vehicles. The intra-platoon control laws can regulate spacing with very high precision but require additional information not available through sensors (such as the deceleration of the vehicle ahead and the deceleration of the leader of the platoon). Therefore a high level of inter-vehicle cooperation is needed within a platoon. Moreover, the laws of [6] require larger and larger control inputs the

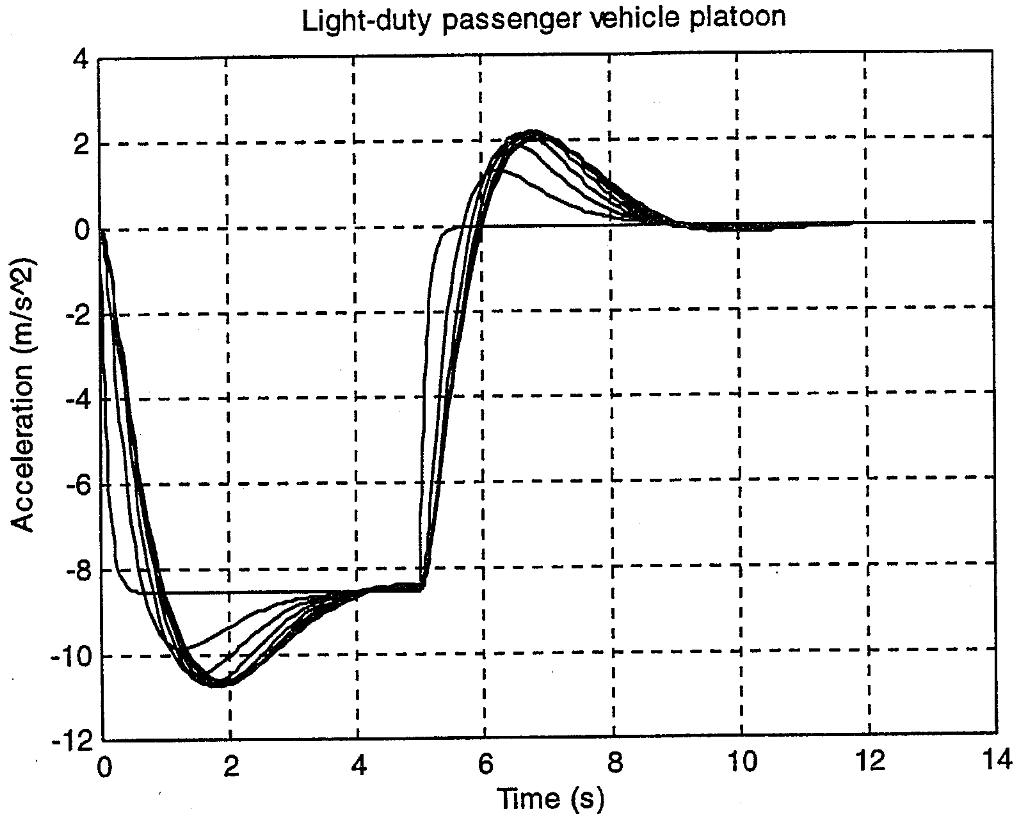

FIGURE 2 Acceleration versus time for a platoon of ten light-duty passenger vehicles with $\tau = 120\,\mathrm{ms}$, and the platoon leader braking at a rate of $8.5\,\mathrm{m/s^2}$.

further back a vehicle is in the platoon (see Fig. 2). This implies that if the leader brakes at the rate $a^{A}_{\min}$ and some follower cannot brake harder than $a^{A}_{\min}$, then intra-platoon collisions can occur.

To avoid such collisions we define the platoon *brake amplification factor*, α, as:

$\alpha =$ maximum peak braking by any follwer/lead vehicle peak braking.

This quantity depends on the design of the intra-platoon controller. For our analysis we pick a well-known, experimentally verified control law from [6] and compute α via simulation. For the chosen law it can be analytically shown [6] that α is independent of the magnitude of lead vehicle braking and that the braking effort stops growing after the first few followers. Figure 2 provides the data necessary to derive α. Observe that the fourth follower brakes the hardest with a peak value of $11\,\mathrm{m/s^2}$. Dividing by $8.5\,\mathrm{m/s^2}$ (the peak value of the leader braking) the value $\alpha = 1.3$ is obtained.

The brake amplification factor can be used to determine the inter-platoon spacing required for safety. The process is the same as in the previous section, with the lead vehicle braking capability modified by the amplification factor α. Consider again the scenario of Fig. 1 and assume that now A and B represent two platoons. The above discussion reveals that if all vehicles in platoon A can decelerate at a rate $a_{\min}^{A}$, then the deceleration of the leader of platoon A should not exceed $a_{\min}^{A}/\alpha$ to avoid intra-platoon collisions. We therefore require that platoon A satisfies the hard braking safety requirement assuming that the leader of platoon A can decelerate at a rate $a_{\min}^{A}/\alpha$ and the platoon B can decelerate at a rate $a_{\min}^{B}$.

Based on the controllers of [6] we can also compute a bound on the intra-platoon spacing needed for safety. The goal of the intra-platoon control law considered above is to maintain constant spacing between adjacent followers. It can be shown [6] that this control law is string stable, that is all vehicles eventually converge to their desired spacing and the maximum of the spacing errors over time decreases as we move further back in the platoon (see Fig. 3). We assume that the lead vehicle brakes at the rate $a_{\min}^{A}/\alpha$ until it comes to rest and compute the spacing error for each follower. Figure 3 illustrates the results. We see that the maximum spacing error over all followers

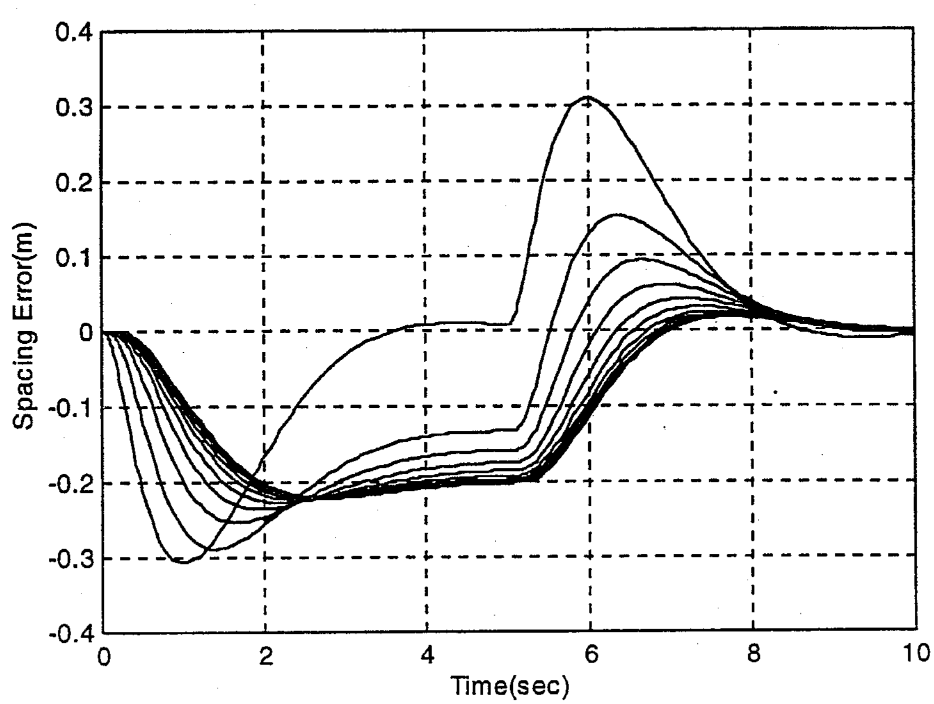

FIGURE 3 Spacing error vs. time for a platoon of 10 passenger vehicles.

and over all times is 0.31 m. Therefore in our calculations we will not allow intra-platoon spacing smaller than this value (in fact the smallest value we consider will be 1 m).

Under the spacing requirements discussed above, collisions can be avoided on a platoon based AHS if the deceleration of the leader does not exceed the value dictated by the brake amplification factor. If this requirement is violated in a platoon (any vehicle in the platoon, except the last follower, braking harder in response to a malfunction or an obstacle for example), then collisions are possible within this platoon. It is conjectured that, because of the tight intra-platoon spacing, all resulting intra-platoon collisions will satisfy low relative velocity safety criterion. Although, it is shown [4] that in a hard braking emergency scenario the first intra-platoon collision will satisfy low-impact velocity safety criterion, further research is required to verify this property for subsequent intra-platoon collisions. Moreover, the intra-platoon collisions in one platoon may require additional inter-platoon spacing to prevent propagation of collisions from one platoon to the next, which implies a reduction in the pipeline capacity for a platoon based AHS. These additional complications are worthy of further research. The interested reader is referred to [8] for more details.

2.3. Pipeline Capacity Equations

Based on the above spacing calculations the pipeline capacity can now be determined. It will depend on the AHS speed, v, and the inter-vehicle spacing. Note that the space, U^{I}, "utilized" by an individual vehicle is given by:

$$U^{\mathrm{I}} = l + D, \tag{3}$$

where l is the vehicle length and D is the inter-vehicle spacing, that is, the distance from the rear bumper of the preceding vehicle to the front bumper of the following vehicle. For steady-state operation, D can be calculated as a function of speed (and the vehicle parameters) using the analysis of Section 2.1. Likewise, the per vehicle space utilization in a platoon, denoted by U^{P}, is given by

$$U^{\mathrm{P}} = \frac{y + nl + (n-1)z}{n}, \tag{4}$$

where y is the inter-platoon spacing, z is the intra-platoon spacing (both bounded by the analysis of Section 2.2) and n is the platoon size (the number of vehicles in a platoon).

The calculations of Sections 2.1 and 2.2 allow us to determine the safe space utilization at steady state as a function of the speed and the vehicle parameters (deceleration capabilities of the subject and preceding vehicles, lags, etc.). The values of these parameters will depend primarily on the vehicle class. Here we consider three vehicle classes: passenger vehicles (cars), buses, and trucks, and assume that all three classes are uniformly distributed along the AHS pipe. Then the pipeline capacity for an AHS containing only individual vehicles is given by

$$C^{\mathrm{I}} = v \Big/ \Big[p_{\mathrm{c}}^2 U_{\mathrm{c,c}}^{\mathrm{I}} + p_{\mathrm{b}}^2 U_{\mathrm{b,b}}^{\mathrm{I}} + p_{\mathrm{t}}^2 U_{\mathrm{t,t}}^{\mathrm{I}} + p_{\mathrm{c}} p_{\mathrm{b}} (U_{\mathrm{b,c}}^{\mathrm{I}} + U_{\mathrm{c,b}}^{\mathrm{I}}) + p_{\mathrm{c}} p_{\mathrm{t}} (U_{\mathrm{t,c}}^{\mathrm{I}} + U_{\mathrm{c,t}}^{\mathrm{I}}) + p_{\mathrm{b}} p_{\mathrm{t}} (U_{\mathrm{b,t}}^{\mathrm{I}} + U_{\mathrm{t,b}}^{\mathrm{I}}) \Big], \tag{5}$$

where v is the vehicle speed, p_i is the percentage of vehicles of class i, $U_{i,j}^{\mathrm{I}}$ is the space utilization (given by Eq. (3)) of a vehicle of class i when it is following a vehicle of class j ($i, j \in \{\mathrm{c}, \mathrm{b}, \mathrm{t}\}$ for car, bus or truck). The pipeline capacity for an AHS that supports platooning can be similarly determined. We assume that platoons consist exclusively of vehicles from a single class. The formula for the pipeline capacity of a platoon based AHS can be obtained by replacing $U_{i,j}^{\mathrm{I}}$ with $U_{i,j}^{\mathrm{P}}$ in Eq. (5). Calculation of $U_{i,j}^{\mathrm{P}}$ (given by Eq. (4)) involves n_i, the nominal size of a platoon of class i, z_i, the nominal intra-platoon spacing for class i and $y_{i,j}$, the inter-platoon spacing if a platoon of class i is following a platoon of class j.

3. NUMERICAL ANALYSIS RESULTS

3.1. Model Parameter Values

In this section we discuss nominal values and ranges used for the parameters used in the analysis.

The capacity calculations require nominal values for the various system parameters: lags, deceleration capabilities, jerk limits, vehicle lengths, etc. Sensitivity of the capacity with respect to variations about the nominal values are investigated for some of the parameters. The interested reader can find further details in [14].

For the vehicle class percentages, we use a typical urban highway mix of 93 percent light-duty passenger vehicles (LDPV), 6 percent trucks and 1 percent buses. The vehicle lengths are assumed to be 5, 12 and 20 m for passenger vehicles, buses, and trucks respectively.

The nominal lag values used to derive minimum safe inter-vehicle spacing are shown in Tables VII and VIII of the appendix to this paper. The lag depends heavily on the degree of inter-vehicle cooperation. It is assumed that autonomous vehicles have to detect any emergency deceleration of the preceding vehicle by filtering sensed velocity data. Therefore the total (lumped) lag for autonomous vehicles is the sum of a sensing (including filtering) and an actuation lag. Low-cooperative vehicles, on the other hand, obtain information about emergency deceleration via communication. Therefore, the total lag for low-cooperative vehicles is the sum of a communication lag and an actuation lag. This lag is assumed to be smaller than the total lag for autonomous vehicles.

For the communication lag itself two different cases are considered. In the first case, vehicle B sends a warning message to vehicle A only when its acceleration reaches $a^{\mathrm{B}}_{\mathrm{min}}$. Therefore the overall emergency communication lag includes a brake actuation lag (for vehicle A) *and* a communication lag. In the second case, vehicle B sends a message as soon as it commands its actuators to apply hard braking. The overall lag is substantially smaller in this case; it consists of a communication lag plus an additional small lag (assumed to be 20 ms here) due to possible differences between the brake actuators of vehicles A and B. Finally, high-cooperative vehicles continuously communicate state information for control purposes, and therefore it is assumed that they can infer emergencies quicker than low-cooperative vehicles. Inter-platoon operation is assumed to be low cooperative.

The maximum permissible jerks for light-duty passenger vehicles (cars), buses, and trucks are assumed to be 7.5, 5, and $3g$/s, respectively. These levels are known to cause discomfort to the occupants of conventional vehicles. The values are supposed to reflect emergency situations in which the requirement for comfort is secondary. They are based only on the requirement that the vehicle occupants not be ejected from the vehicle during hard braking.

Our analysis also requires the maximum braking rates for the three vehicle classes. Maximum braking rates for 1995 models for new light-duty passenger vehicles on dry road surfaces are used as a starting point to create a braking capability distribution [5]. We use data reported in *Road and Track* for the braking capabilities of specialty vehicles, while relying on data reported in *Consumer Reports* for all other types of light-duty passenger vehicles. In order to populate the distribution, we used the North American production figures (as reported in *Automotive News* [10]) for each model in the sample for the time period January 1 through April 15, 1995. For validation purposes, the test track data values were compared with the minimum braking rates specified in *Federal Motor Vehicle Safety Standard Number 105: Hydraulic Brake Systems*. We also made an allowance for degradation of braking performance due to damage and normal wear of brake system components over the useful life of the vehicle, changes in the tire condition, and adverse environmental conditions (such as rain and snow). We assume that the vehicle population for each model is uniformly distributed over the range 100 to 70 percent of the braking capability for a new vehicle of that model braking on a dry road. The resulting distribution is shown in Fig. 4.

For buses, the maximum braking is calculated based on the operating profile specified in guidelines for the design of transit coach vehicles for the high-speed duty cycle [11]. For trucks, the nominal values used are based on a synthesis of test track results published by the National Highway Traffic Safety Administration Vehicle Test Center [12]. The test track results are reported in terms of required stopping distance for an initial speed of 60 mph from which the truck applies maximum braking (see Table IX in the Appendix). The values are within the range specified in *Federal Motor Vehicle Safety Standard Number 121: Air Brake Systems*. We found that it is not enough to search for the braking performance of a particular heavy vehicle model. When a bus or truck is purchased, the buyer must select among different braking, power, and transmission systems. Thus, two heavy vehicles of the same model can have different performance characteristics. Table I is constructed from the braking data and provides the range of maximum deceleration capabilities for each class of vehicles.

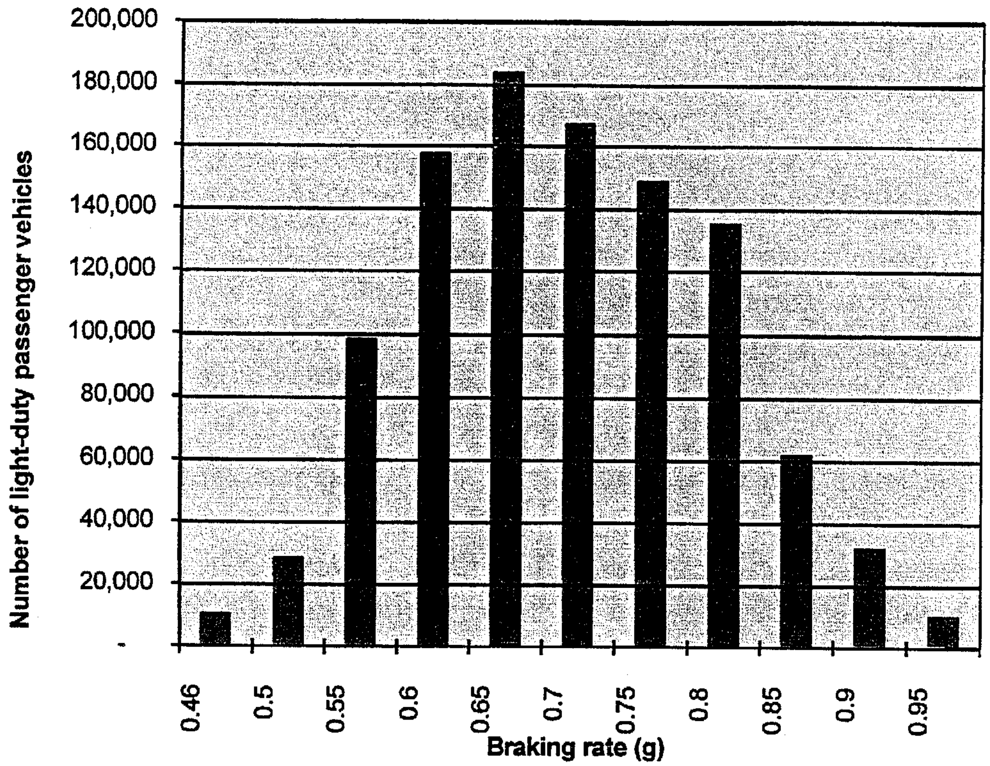

FIGURE 4 Discrete probability distribution of light-duty passenger vehicle braking rates.

TABLE I Minimum and maximum values for the range of deceleration rates

Vehicle class	*Minimum deceleration rate (g)*		*Maximum deceleration rate (g)*
	Individual vehicles	*Platoon leaders*	
Passenger vehicles	0.46	0.46/1.3	0.98
Buses	0.20	0.20/1.5	0.54
Trucks	0.26	0.26/1.5	0.54

The deceleration values in Table I are used in the safe spacing calculations as follows. For any vehicle pair, the deceleration capability of the rear vehicle is picked from column 1 and the deceleration capability of the front vehicle is picked from column 2. The row is determined by the vehicle class. For example, if a truck is following an LDPV, then the assumed deceleration capability of the truck would be $0.26g$ and that of the LDPV would be $0.98g$. For a pair of platoons, Table I is used similarly. As shown in Table I, the deceleration capability of the leader of the rear platoon is derated by the appropriate amplification factor from Table II (discussed next).

TABLE II Intra-platoon spacing and brake amplification factor at 30 m/s

Vehicle class	*Intra-platoon spacing* (m)	*Braking amplification factor*
Light-duty passenger vehicles	2	1.3
Buses	8	1.5
Heavy articulate trucks	8	1.5

For steady-state conditions the relative velocity between vehicles should nominally be zero. This will not always be the case, however, due to errors in desired speed tracking. To make our results more realistic, we introduce some uncertainty in the relative velocity. We assume that the maximum value of the speed tracking error during normal operation is 1.5 percent [13] and assume, that the following vehicle travels 1.5 percent faster than the preceding vehicle in the spacing design.

Finally, the spacing between two individual vehicles and between two platoons is calculated in Sections 2.1 and 2.2 using the hard braking safety criterion. This spacing may be extremely small when the maximum braking rate of the following vehicle is much greater than that of the preceding vehicle (e.g., a car following a truck). To avoid the use of unrealistic spacing values, we impose an additional constraint. We assume that the trailing vehicle follows at a distance equal to the length of the leading vehicle whenever this value happens to be greater than the spacing calculated by the hard braking safety requirement. The only two cases when this restriction is relevant are when a car follows a bus or truck.

3.2. Spacing and Pipeline Capacity

Using the above parameters and the techniques discussed in Section 2, we calculate values for the safe spacing and use them to infer values for the pipeline capacity. The minimum intra-platoon spacing and the brake amplification factor are calculated by a simulation tool described in Section 2.2. The tool simulates the response of the vehicles in a platoon to a hard braking disturbance generated by the lead vehicle. Each vehicle in the platoon applies the control law developed in [6] to satisfy string stability requirements. The simulation results confirm that when the maximum braking of the platoon leader is limited by the application of the derating factor α to the worst braking capability within a platoon, there will be no intra-platoon collisions. Table II shows the nominal braking amplification factors and intra-platoon spacing values. The maximum intra-platoon spacing error turns out to be very sensitive to the intra-platoon communication delay. For light-duty passenger vehicles the spacing error doubles (from approximately 0.5 to 1 m) as the communication delay increases from 20 to 100 ms. Therefore, we use 2 m intra-platoon spacing (which is safe up to a 200 ms lag) in the rest of the analysis. In contrast, the maximum braking amplification factor is not very sensitive to communication delay: as the communication delay increases from 20 to 100 ms, the maximum braking amplification for platoons of light-duty passenger vehicles increases by approximately 0.02 percent.

The inter-individual-vehicle and inter-platoon nominal spacing is calculated using the parameter values given above. Table III contains the results of our analysis for an operating speed of 30 m/s (67 mph). The nominal values of lags, braking capabilities, amplification factors, and jerks are as previously discussed.

TABLE III Inter-vehicle and inter-platoon spacing at 30 m/s

Preceding vehicle [following vehicle]	*Autonomous individual vehicle* (m)	*Low-cooperative individual vehicle* (m)	*High-cooperative individual vehicle* (m)	*Inter-platoon* (m)
Light-duty passenger vehicle [light-duty passenger vehicle]	58	57	55	86
Bus [bus]	153	152	151	266
Truck [truck]	103	102	100	189
Bus [light-duty passenger vehicle]	21	20	18	49
Truck [light-duty passenger vehicle]	21	20	20	49
Light-duty passenger vehicle [truck]	140	139	137	226
Light-duty passenger vehicle [bus]	190	189	187	303
Truck [bus]	153	152	150	266
Bus [truck]	103	102	101	189

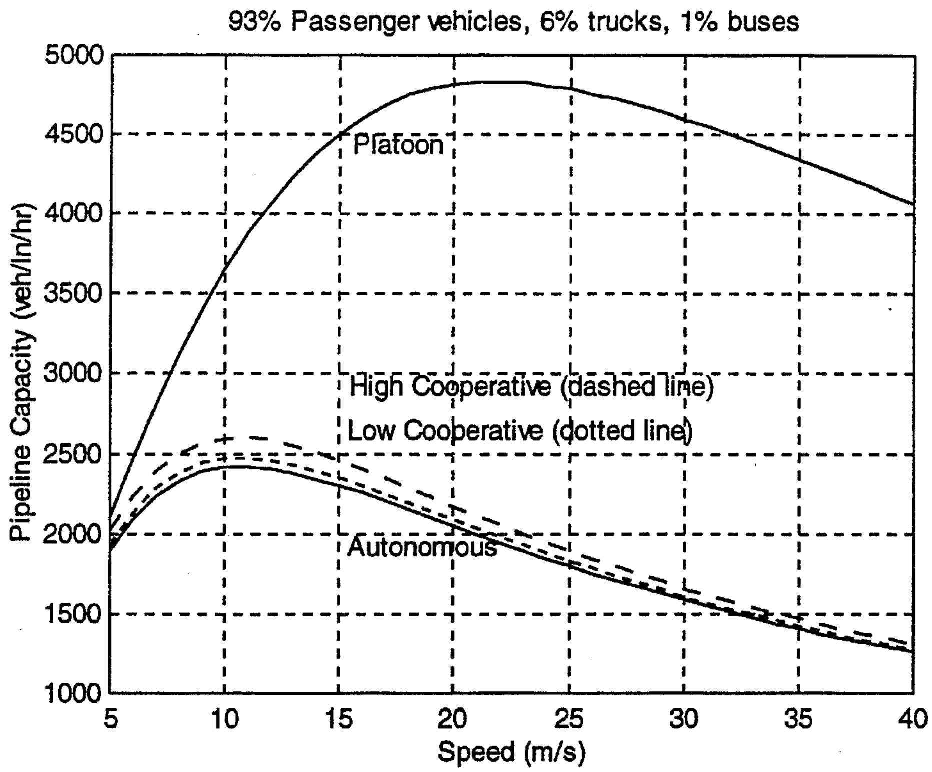

FIGURE 5 Pipeline capacity as a function of speed for a typical urban vehicle class mix.

The spacing numbers are combined to calculate pipeline capacity for the nominal vehicle class mix. Figure 5 shows pipeline capacity as a function of operating speed. The platooning plot corresponds to nominal platoon sizes of ten LDPV, three buses, and two trucks. These values are used as a base case about which the sensitivity of the capacity with respect to variations in the system parameters is investigated.

3.3. Sensitivity Analysis

3.3.1. Inter-Vehicle Cooperation, Operating Speed and Lag

Figure 5 shows that *large platoons produce much higher pipeline capacity than individual vehicles.* Moreover, *as the level of inter-vehicle cooperation increases* (as we go from autonomous to low- and high-cooperative vehicles), *there is a corresponding increase in pipeline capacity across the range of speeds.* The estimation of actual platoon throughput from pipeline capacity is complicated by additional losses due to platoon formation and dissipation. We also observe that knowledge of acceleration of the preceding vehicle via communication does not provide significant benefit, as seen from the difference in capacity of the high- and low-cooperative individual vehicle based AHS. The following figure explains this in more detail.

Figure 5 also indicates that the capacity is sensitive to variations in operating speed. *For all AHS designs, as the speed increases the pipeline capacity attains a peak value and then gradually decreases.* The peak occurs at higher speeds for platoons than for individual vehicles. These conclusions hold even if the lags and nominal braking capabilities are varied about the nominal values. Further details may be found in [14].

Figure 6 shows the affect of variations in the vehicle sensing and actuation lags on pipeline capacity. The plot is for the case of low-cooperative individual vehicles on a highway populated entirely by LDPV. The pipeline capacity changes approximately five percent

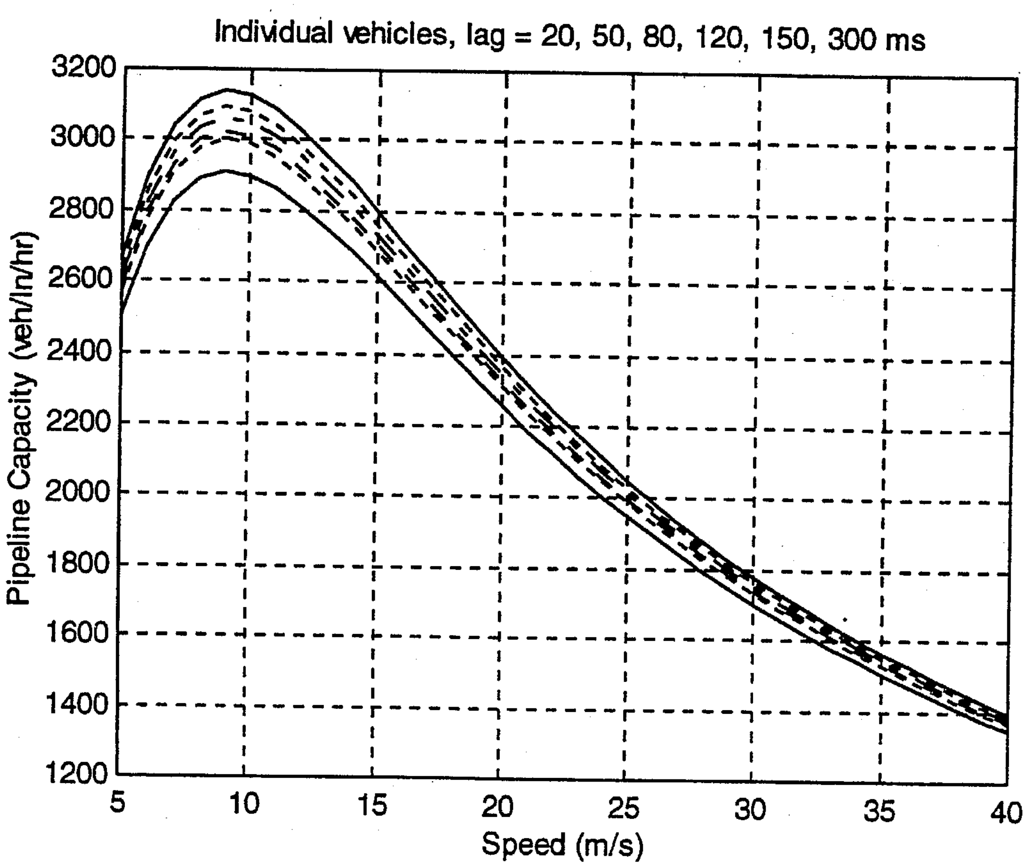

FIGURE 6 Pipeline capacity vs. speed for different values of lumped lag, individual LDPV.

between the lowest and highest lags, indicating *low sensitivity of pipeline capacity to lags.* The lag variation from high to low cooperative falls within this range.

3.3.2. Vehicle Class Mix

To test the effect of vehicle class mix, we vary the ratio of trucks to LDPV, from zero to 100 percent. Figure 7 shows the variation in pipeline capacity for platoons and high-cooperative individual vehicles at 30 m/s. As the ratio of trucks increases, the pipeline capacity decreases. Furthermore, as the ratio of trucks increases, the rate of decrease (i.e., the slope) of the pipeline capacity curve becomes smaller. This is due to the fact that the inter-vehicle spacing between two trucks and between a truck following an LDPV is much greater than the one between two LDPV. This in turn is due to the fact that light-duty passenger vehicles can brake at much higher rates than

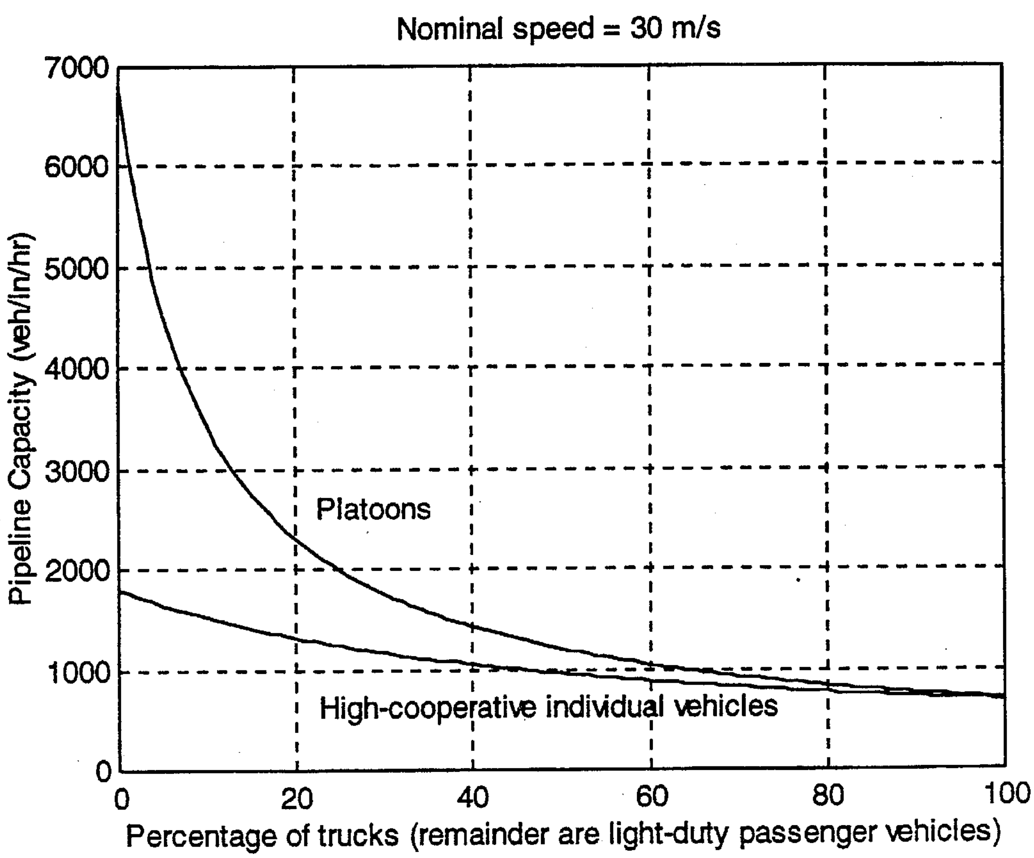

FIGURE 7 Sensitivity of capacity to class mix for individual and platooned operation.

trucks. The first 10 percent of trucks produce a large reduction (49 percent) in the capacity of a platooned AHS, and then the rate of reduction becomes less pronounced. For individual vehicles there is a decline in pipeline capacity between 15 percent and 16 percent as the ratio of trucks increases from zero to 10 percent. These observations are also confirmed for low cooperative and autonomous vehicles and for speeds between 5 and 40 m/s (see [14]).

In Summary, the results indicate that *the pipeline capacity is very sensitive to the proportion of heavy vehicles in the AHS vehicle population*. The sensitivity is much higher for platoon operation.

3.3.3. Platoon Length and Intra-Platoon Spacing

First we vary the platoon length from one to ten vehicles for each of the vehicle classes, with a step size of one vehicle. The result for the LDPV case is shown in Fig. 8. The capacities for buses and trucks

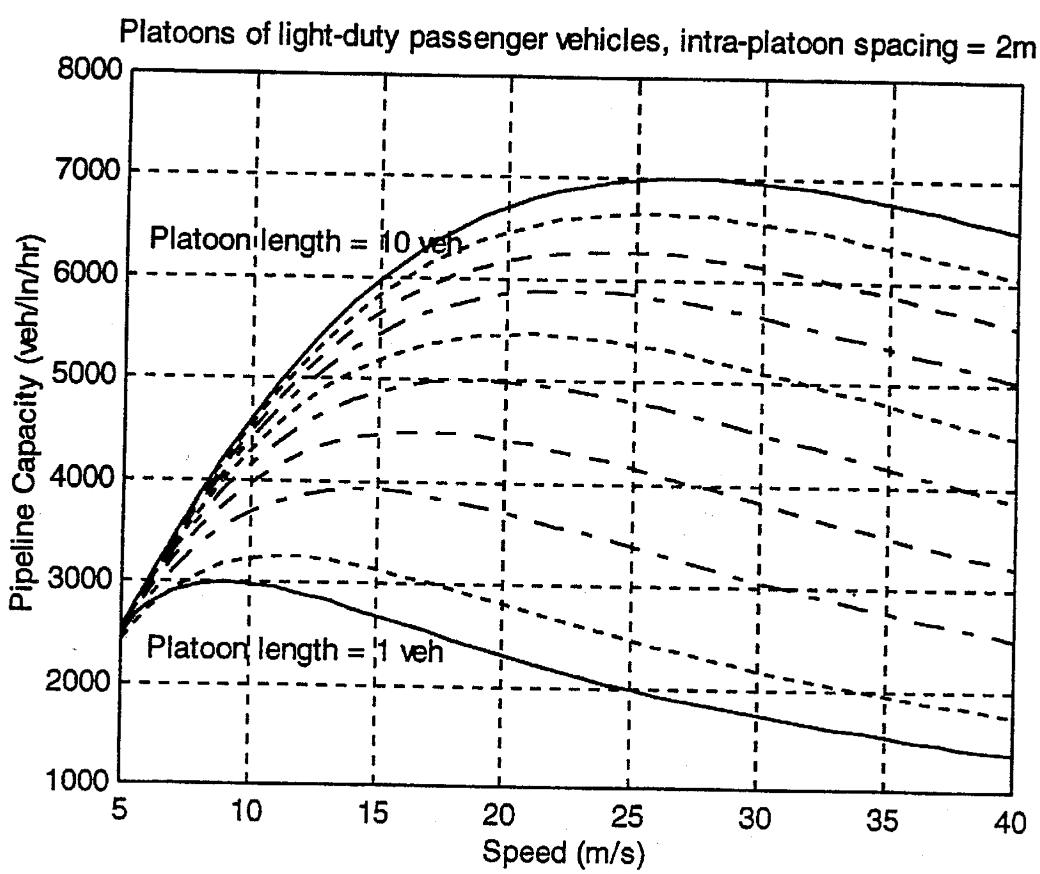

FIGURE 8 Pipeline capacity versus platoon length for LDPV.

exhibit the same trends, although the scales of the ordinate axes are different. *Pipeline capacity increases as the platoon length increases.* Moreover, the speed at which the peak pipeline capacity is obtained increases as the platoon length increases while the rate of decay decreases after the peak.

Next, we investigate the effect of varying the intra-platoon gap from 1 to 15 m, while holding the platoon size constant. The nominal platoon sizes for passenger vehicles, buses, and trucks are ten, three, and two, respectively. The results for LDPV and buses are shown in Figs. 9 and 10. *Pipeline capacity decreases as the intra-platoon spacing increases*. The truck case behaves similarly (see [14]).

3.3.4. Vehicle Braking Capability and Check-in Policy

Our analyses show that the wide variation in braking capability amongst the vehicle population has a significant negative impact on

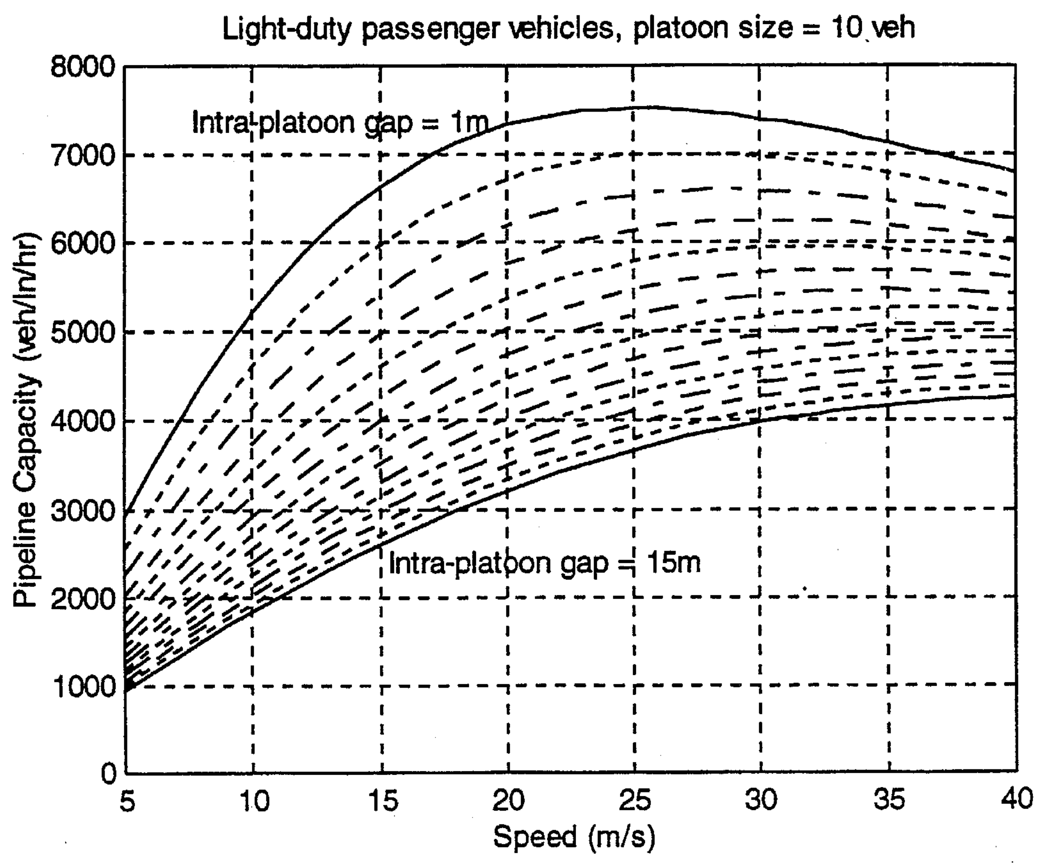

FIGURE 9 Pipeline capacity versus intra-platoon spacing for LDPV.

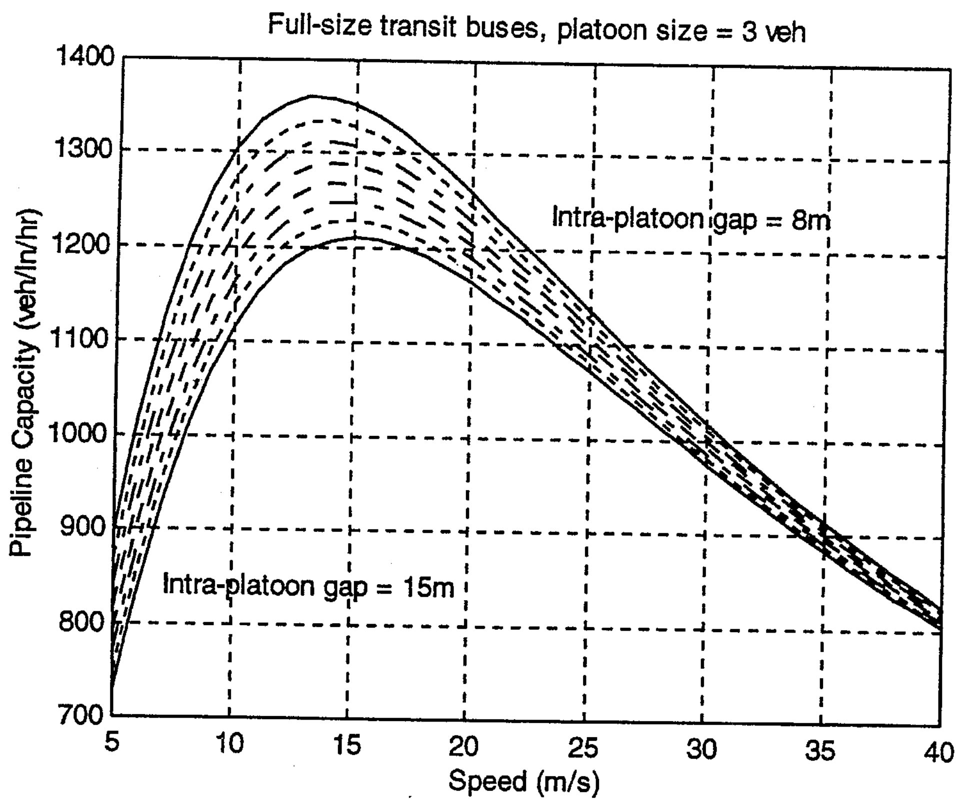

FIGURE 10 Pipeline capacity versus intra-platoon spacing for buses.

capacity. In this section, the capacity effects of a check-in policy that refuses entry to vehicles with deceleration capabilities below a certain level is investigated. A check-in policy may be evaluated by its associated pipeline capacity and *coverage*. Coverage refers to the percentage of the vehicle population admitted by the check-in policy. One would expect that decreasing coverage increases capacity. The trade-offs are quantified in this section. The analysis is restricted to LDPV-only AHS. We do not have data on the statistical variations in the braking capabilities of heavy vehicles.

Table IV shows the coverage values used in the analysis. The minimum and maximum values of the braking capability associated with the coverage are shown. In every case the maximum stays fixed at 0.98g. The minimum braking capability used in the spacing design varies as the coverage is raised by lowering the threshold on the acceptable level of braking capability.

The results for the high-cooperative and platoon cases are shown in Figs. 11 and 12. The results for the autonomous and low-cooperative vehicle cases are similar (see [14]). Table V summarizes the results of

TABLE IV Percentage difference in low and high braking rates

Percentage of vehicle braking distribution, permitted by check-in, as measured from the right tail to the left	*Lowest braking rate (g), light-duty passenger vehicle*	*Highest braking rate (g), light-duty passenger vehicle*
71	0.65	0.98
87	0.60	0.98
96	0.55	0.98
99	0.50	0.98
100	0.46	0.98

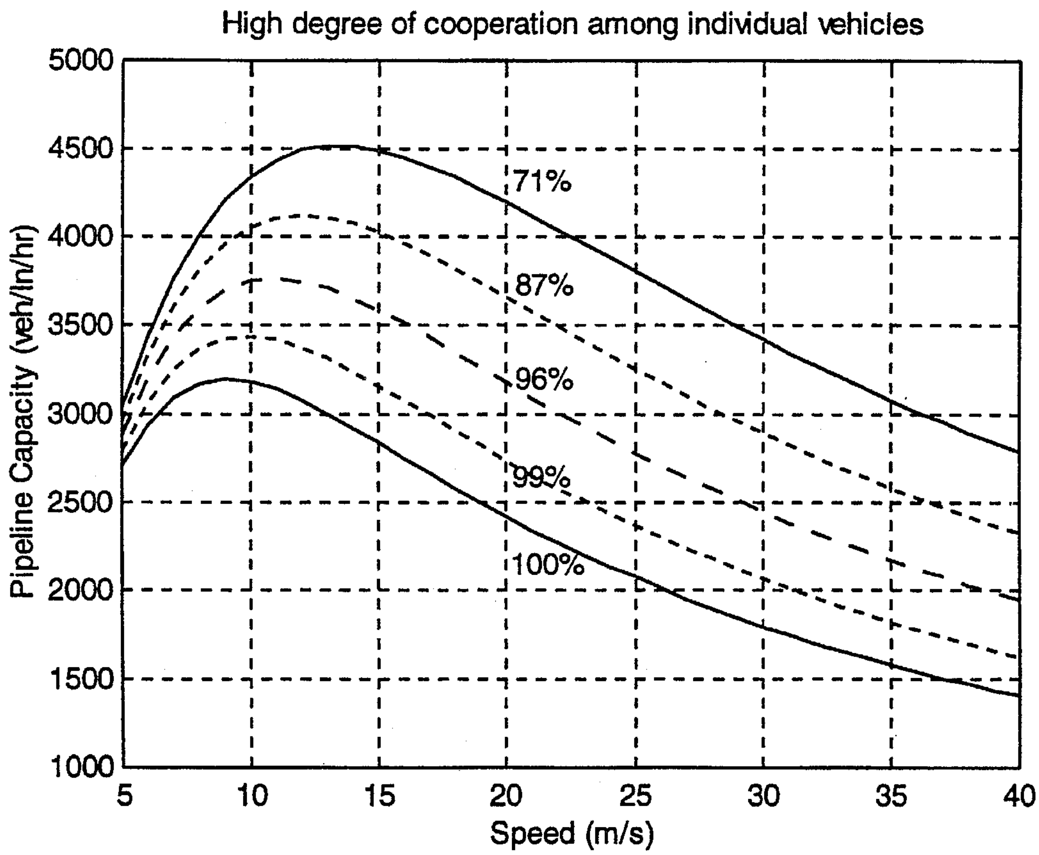

FIGURE 11 Sensitivity of pipeline capacity to coverage for high-cooperative individual vehicles.

the test cases. *For all speeds, pipeline capacity increases as the coverage decreases.* The reason is that, as the number of vehicles with lower braking-rate capabilities increases within the pipeline, there is a corresponding increase in the minimum permissible inter-vehicle and inter-platoon spacing, which translates into a higher space utilization and lower capacity.

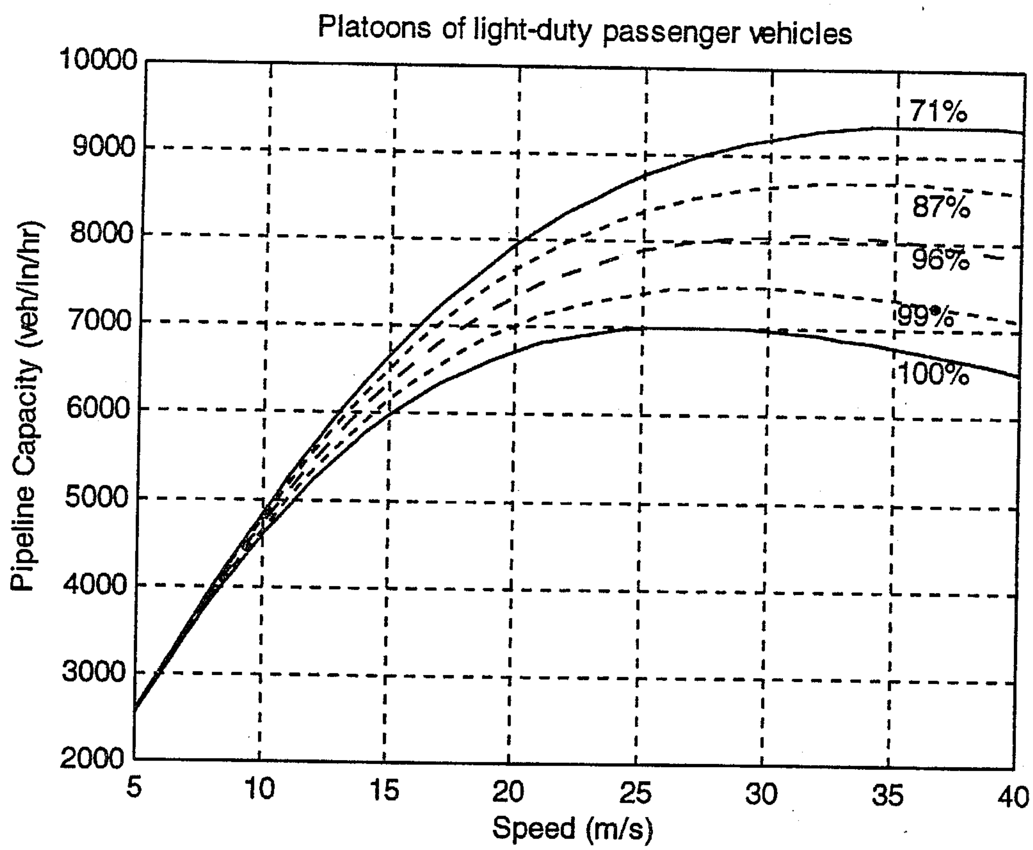

FIGURE 12 Sensitivity of pipeline capacity to coverage for LDPV platoons.

TABLE V Pipeline capacity at 30 m/s for different check-in requirements

Distribution of intelligence	*Pipeline capacity* (veh/ln/hr)				
	Percentage of vehicle braking distribution permitted by check-in, as measured from the right tail to the left				
	100%	99%	96%	87%	71%
Autonomous individual vehicle	1702	1939	2270	2644	3069
Low-cooperative individual vehicle	1732	1980	2329	2728	3188
High-cooperative individual vehicle	1795	2064	2447	2892	3415
Platoon	6973	7467	8064	8637	9188

As the strictness of the check-in policy increases (i.e., from allowing all vehicles to enter the AHS lanes to disallowing vehicles in the left tail of the distribution), the pipeline capacity increases. The magnitude of the increase varies between 80 percent and 90 percent for the individual vehicle cases, while the increase for the platoon case is 32 percent. The increase for platoons is smaller because, using the low impact safety criterion, the intra-platoon spacing is independent of the width of the braking distribution (it depends only on the maximum braking capability). We conclude that *pipeline capacity is highly sensitive to coverage.*

3.3.5. Comparison of Uniform and Non-Uniform Spacing Design

In the previous section we saw that the pipeline capacity is highly sensitive to the wide variation in braking capability. The inter-vehicle spacing for every vehicle pair is picked using the lowest and highest values of the distribution to obtain safety in the hard braking sense. Since the same deceleration values are used for all vehicle pairs, in many cases the spacing is highly conservative. For example, if we pick a vehicle pair at random from an AHS lane, the likelihood that the braking capability of the front vehicle is the maximum of the distribution and that of the rear vehicle is the minimum of the distribution is relatively low.

In this section we explore an alternative AHS spacing design idea that promises significant capacity benefits. If each vehicle can estimate its own braking capability, it can follow the vehicle ahead at a safe separation distance for its actual braking capability (rather than the minimum of the distribution). This can reduce inter-vehicle spacing for autonomous vehicles. In addition, if each vehicle also obtains (via communication) the braking capability of the preceding vehicle, the inter-vehicle spacing may be further reduced for cooperative vehicles. Such a spacing design, where inter-vehicle spacing values are not necessarily the same for all vehicle pairs of the same class on the highway, is called a *non-uniform spacing* design.

We compare the pipeline capacities resulting from uniform and non-uniform spacing policies. Figure 13 shows that if individual low-cooperative vehicles operate under a non-uniform spacing policy, the resulting pipeline capacity far exceeds that afforded under uniform spacing for speeds between 8 and 40 m/s. The distance between the pipeline curves corresponding to uniform and non-uniform spacing policies initially increases with speed and then becomes constant.

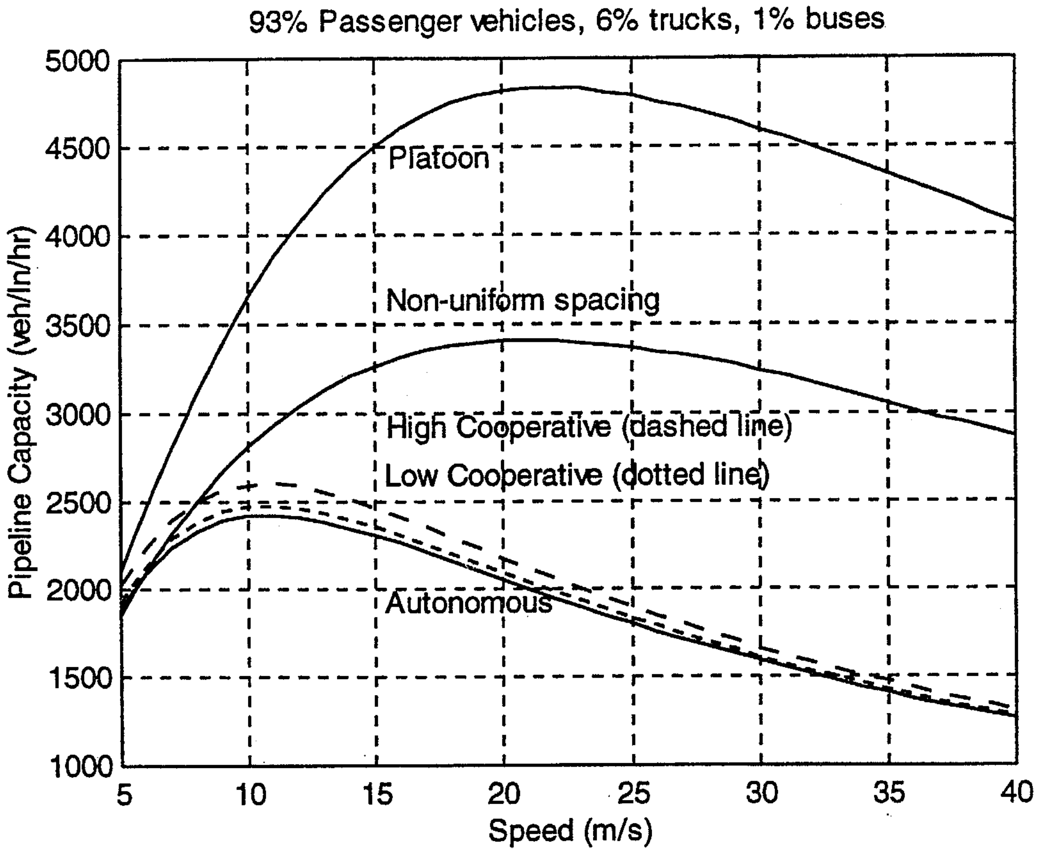

FIGURE 13 Pipeline capacity vs. speed for non-uniform and uniform spacing policies.

The non-uniform spacing calculation is performed as follows. We assume that each vehicle knows its own braking capability within a range of $0.05g$. The braking distribution of Fig. 4 is divided into 10 bins each having a width of $0.05g$. For all vehicles within a bin, a worst case design (i.e., worst braking rate of the bin for the following vehicle and best braking rate of the bin for the leading vehicle) is used to obtain average spacing. As the spacing design is highly sensitive to differences in the braking rates of the two vehicles, very small spacing (of the order of a few meters) is possible for certain combinations of braking rates. It is not clear whether such small spacings can be maintained comfortably during the normal mode of operation without communicating additional information (as is done in platooning). This issue requires further research. We therefore impose a minimum headway of 0.5 s (spacing of 0.5 times velocity) on any vehicle following according to non-uniform spacing design. This arbitrary bound on the minimum time separation of 0.5 s between non-uniform spaced vehicles results in lower capacities than the uniform spacing design at speeds below 8 m/s.

At the nominal speed of 30 m/s, the difference in pipeline capacity between uniform spacing platoon operation and cooperative individual vehicle operation under non-uniform spacing is approximately 29 percent. The difference between platoon operation and individual vehicles with uniform spacing is much greater, as shown in Table VI. We therefore conclude that *the availability of information about the braking capabilities can result in a substantial increase in pipeline capacity.*

3.3.6. Combined Effect of Reduced Braking Capability Variation, Lags, and Non-Uniform Spacing

In this section, we analyze the combined effect of various parameters on pipeline capacity for an AHS consisting of LDPV only. We observe that (Fig. 14) non-uniform spacing design with reduced lags and reduced variation in braking capabilities provides a substantial increase in capacity of the individual vehicle case. The effects of these three capacity-enhancing modifications are not additive because the reduction in variation of braking capabilities means that the non-uniform spacing design offers less of an advantage. Figure 14 also shows that the capacity is not very sensitive to the variation between 70 and 150 ms in the total lag. Thus, the effect of reducing lags is not substantial.

On the other hand, the capacity is very sensitive to the variability in the braking capabilities of vehicles, particularly when uniform

TABLE VI Percentage reduction in pipeline capacity at 30 m/s (platoon capacity is 100%)

Level of cooperation	*Spacing policy*	*Reduction in pipeline capacity from platoon operation (percentage of platoon capacity)*
Autonomous individual vehicle	uniform	66
Low-cooperative individual vehicle	uniform	64
High-cooperative individual vehicle	uniform	63
Cooperative individual vehicle	non-uniform	29

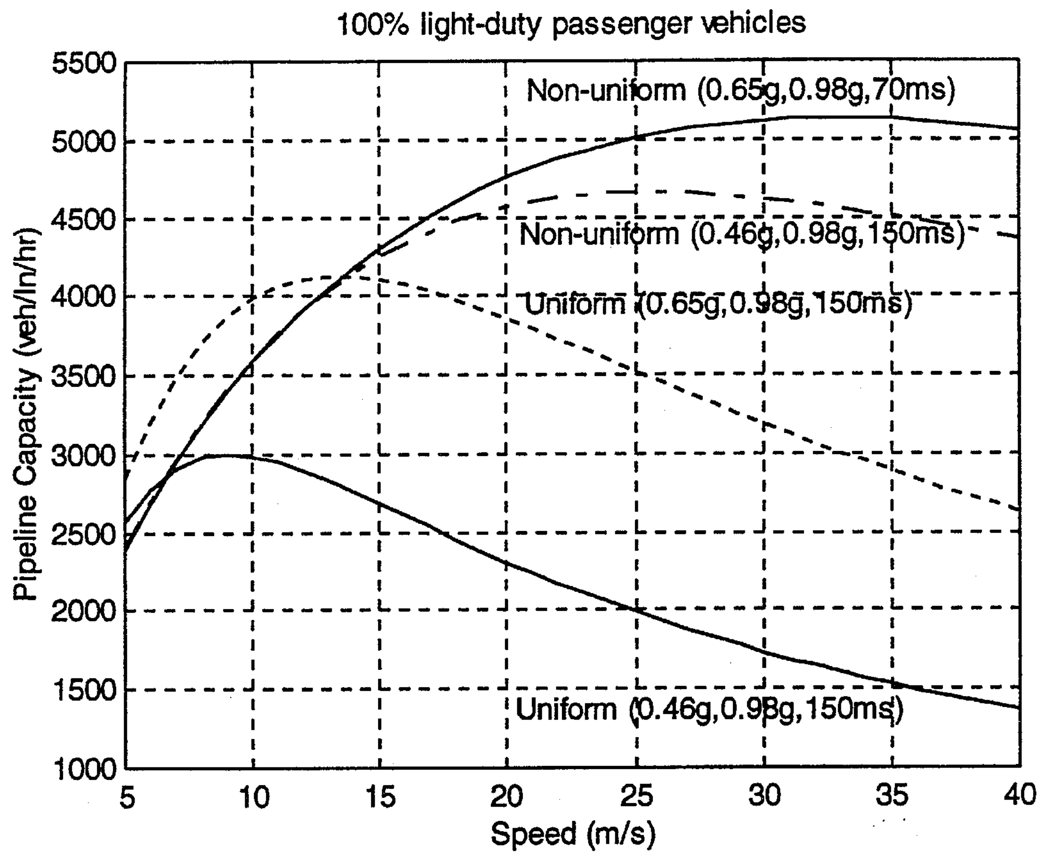

FIGURE 14 Comparison of pipeline capacities for non-uniform and uniform spacing policies, given wide and narrow ranges of braking rate capabilities and different values of lumped lags.

spacing policies are applied. If the entire braking rate distribution is permitted (0.46*g* for follower and 0.98*g* for leader), the pipeline capacity at 30 m/s is only 1700 vehicles per hour. When this distribution is truncated at the low end (0.65*g* for follower and 0.98*g* for leader), pipeline capacity increases to about 3200 vehicles per lane per hour.

Cooperative non-uniform spacing can permit significantly higher capacities than uniform spacing, but the advantage decreases as the braking distribution is truncated. If the entire braking distribution is permitted, non-uniform spacing permits a 180 percent increase in pipeline capacity at 30 m/s, but if the narrowest of the two braking distributions is used, this increase is only about 60 percent (but relative to a much larger starting value).

4. SUMMARY AND FUTURE WORK

We presented a methodology to calculate minimum safe inter-vehicle spacing for an AHS pipe as a function of vehicle capabilities and the information available for longitudinal control. The spacings were used to calculate an upper bound on the per-lane capacity of an automated highway system over a range of operating speeds. The spacing design is also useful for obtaining safe operating parameters for lane change, entry, and exit maneuvers [9,19]. The resulting inter-vehicle spacing values form the basis for the estimation of AHS throughput [2,3,21]. For the modeling assumptions and range of parameter values used in this analysis, one can conclude the following:

- AHS pipeline capacity increases as the degree of inter-vehicle cooperation increases.
- As highway speed increases, AHS pipeline capacity peaks and then decreases.
- AHS pipeline capacity decreases as the heterogeneity of the vehicle mix increases.
- AHS pipeline capacity increases as platoon length increases.
- AHS pipeline capacity decreases as intra-platoon spacing increases.
- AHS pipeline capacity increases as stricter braking capability requirements are imposed for entry to the AHS.
- Adjusting vehicle spacing based on real-time estimation of the vehicle braking capabilities increases AHS pipeline capacity.
- These trends are preserved when we combine variations of different parameters, but the effects are not additive.

The analysis stresses the importance of inter-vehicle coordination and knowledge of a vehicle's braking capability. Three distinct levels of capacity emerge for different AHS designs. An AHS consisting of individual vehicles with uniform spacing produces comparable pipeline capacity to manual traffic (although with improved safety [4]). The same AHS with real-time estimation of the vehicle's braking capability can produce substantially higher capacities. This suggests the need for research in the area of real-time estimation of braking capability. Finally, the capacities obtained for platoons of up to ten LDPV can be even higher.

Even though the inter-vehicle and inter-platoon spacing is designed to ensure that no collisions occur, inter-vehicle collisions can still arise in the event of hard braking in the following cases:

- In the case of non-uniform spacing design, collisions can occur due to an error in the estimation of braking capability.
- In the case of platooning, intra-platoon collisions may result from hard uncoordinated braking in case of either (i) a mismatch between the braking capabilities of the followers or (ii) control loop lags.
- Collisions can arise in the case of inconsistencies in the pavement surface (e.g. oil spills or ice patches).

The safety analysis in [4] shows that in the case of a two vehicle platoon, such intra-platoon collisions will be of low severity (impact velocity less than 3 m/s). There is some work on the multiple collisions for larger platoons [8,20]. The literature indicates that as *the platoon size increases, the severity and number of collisions per platoon increase*, if all vehicles in a platoon slam their brakes in response to hard braking by the platoon leader. The extent of the increase in collision frequency and severity is sensitive to assumptions about the collision dynamics. Unfortunately, such dynamics are still poorly understood. Moreover, platoons are equipped with high-performance communication systems that allow tightly coordinated braking. The impact of such coordinated emergency response strategies on subsequent collisions is also not well understood. These unresolved issues merit further research. Similar studies are required to ascertain the safety impact of braking capability estimation errors. The effect of minimum headway for non-uniform spacing designs also needs to be quantified for ride comfort. Findings from such analyses may indicate that the capacity analyses need to be revisited. The methods used in this paper can also be applied to spacing design criteria that incorporate inter-vehicle collisions (see [9]).

The estimation of actual throughput is also an interesting topic of research. The pipeline capacity is unlikely to be realized by an AHS lane due to transient behaviors such as lane changes, entry, exit and platoon formation and dissipation. It is desirable that the degradation in pipeline capacity be related to parameters such as entry rates, exit rates, and platoon size. Finally, the effect of faults and adverse environmental conditions on the safety and capacity of an AHS and the interface between the AHS and the urban street network are important areas on which research is needed.

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APPENDIX: PARAMETER TABLES

TABLE VII Nominal lags and lumped lags for individual light-duty passenger vehicles

Type of lag	*Autonomous individual vehicle*	*Low-cooperative vehicle*	*High-cooperative vehicle*
Sensing	200 ms	NA	NA
Communication	NA	50 ms	20 ms
Actuation	100 ms	100 ms	100 ms
Lumped (i.e., total)	300 ms	150 ms	120 ms
Lumped, with overlapping actuation and communication delays	300 ms	70 ms	40 ms

TABLE VIII Nominal lags and lumped lags for individual buses and trucks

Type of lag	*Autonomous individual vehicle*	*Low-cooperative vehicle*	*High-cooperative vehicle*
Sensing	200 ms	NA	NA
Communication	NA	50 ms	20 ms
Actuation	1 s	1 s	1 s
Lumped (i.e., total)	1.2 s	1.05 s	1.02 s

TABLE IX Braking distances and rates for trucks

Type of truck	*Worst stopping distance* (ft)	*Worst braking rate* (*g*)	*Best stopping distance* (ft)	*Best braking rate* (*g*)
Bobtail tractor	463	0.26	233	0.52
Empty tractor/trailer	319	0.39	225	0.54
Loaded tractor/trailer	273	0.44	230	0.52